

2. Consumer Liquidity Demand and Government as Lender of Last Resort

Abstract

I consider the role of the government as a Lender of Last Resort in the general equilibrium of an economy with financial intermediation in which financial intermediaries transform liquid consumer deposits to illiquid long-term investment loans to entrepreneurs. In an equilibrium with no default, an endogenous liquidity constraint must be fulfilled at all times, restricting the amount of investment in the illiquid asset that the financial intermediary can make. When the liquidity constraint is binding, there is a role for a Lender of Last Resort who quickly dispatches liquidity at the discount window, helping the financial intermediaries to match current liabilities with maturing assets. The role of the lender of last resort is to help the economy return faster to the long-run equilibrium after an adverse shock relative to the laissez-faire equilibrium. The lender of last resort, moreover, plays the role of keeping the financial-intermediation system active in the face of an adverse shock so large that in a laissez-faire equilibrium would result in the financial system closing down. Government lending of funds at the discount window may involve a "penalty rate", when the shadow value of liquidity for financial intermediaries is sufficiently high.

1 Introduction

The recent financial crisis, as well as the European-countries sovereign debt crisis that followed it, brought into attention the role of a Lender of Last Resort to the financial system. Although significant differences exist between the traditional view of a Lender of Last Resort and the Central Bank interventions during the latest financial crisis (eg Humphrey, 2010), the unconventional monetary policies followed after the crisis of 2008 may be seen as a form of large-scale Lender-of-Last-Resort policy. The traditional view on the Lender of Last Resort stems from the fragility of a fractional reserve banking system and the possibility of banking panics in the absence of a guarantor of the convertibility of deposits to currency. There are different views on the need of a Lender of Last Resort and the form of the Lender-of-Last-Resort intervention (Bordo (1990)). Kindleberger and Aliber (2005) show how the practise of a Lender of Last Resort developed primarily through the monetary policy of the Bank of England without reference to some theory. The monetary policy practise gave rise to the classical view (Thornton (1802), Bagehot (1873)) which mandates that the Central Bank should lend freely, but at a high or (as it has come to be interpreted) at a penalty rate, to everyone with good collateral, preventing illiquid but solvent banks from failing. Friedman and Schwartz (1963) argue that incidence of the Great Depression was due to Fed's failure to act as a Lender of Last Resort during the 1929-1933 contraction. Schwartz (1986) argues that all the important financial crises in the UK and the USA happened when the Central Bank failed to display determination at the beginning of crisis to act as an Lender of Last Resort. Minsky (1986) considers the Lender of Last Resort as the second of the two important

pillars of financial stability in an financial-instability prone economy through socializing costs in times of crisis with the purpose of providing a floor to asset prices and preventing debt deflations. Goodfriend and King (1988) argue that the Lender-of- Last-Resort function of the Central Bank should take solely the form of open market operations (monetary policy), increasing the total quantity of high-powered money, and not the form of discount-window lending to particular financial institutions (banking policy), which requires monitor and supervision and could be provided by private lines of credit; the government provision of such services at a lower cost than the market is dubious. Goodhart (1985, 1987) supports temporal Central Bank assistance to insolvent banks, arguing that the distinction between illiquidity and insolvency is just a myth, as banks asking for Lender-of-Last-Resort support on the grounds of illiquidity will in most cases be under suspicion of insolvency. On the other hand, Meltzer (1986) argues that the Central Bank must allow insolvent banks to fail for not doing so would encourage greater risk taking. Proponents of free banking (eg. Dowd (1988), Gorton (1985), Gorton and Mullineaux (1987), Selgin (1988, 1990)) deny the need for a government Lender of Last Resort, attributing the instability of the financial system to government restrictions on banking and the government-endorsed monopoly in the issue of currency. Calomiris (1993) presents an eclectic view according to which discount-window policy as a means of preventing bank panics is unnecessary and costly, but can have a role in providing occasional and temporal support to particular financial markets, as when confusion arises over the incidence of bad news, or when a reduction in the net worth of financial intermediaries takes place. Bernanke and Gertler (1987) present a macroeconomic version of the model of Diamond and Dybvig (1983), in which the fragile nature of the financial intermediation system under fractional banking gives rise to a "genuine" liquidity constraint

for an equilibrium without default (or bank run).

In the classical view, injections of base money was the essence of Lender-of-Last-Resort operations as a response to a banking panic that could lead to a contraction of the stock of money. In contrast, during the recent crisis Central Banks put their emphasis on unblocking the supply of credit. Before the demise of Lehman Brothers the FED policies were focused on facilitating lending conditions of the interbank market; later a greater focus was placed on direct lending in markets. In both cases the emphasis was on activating frozen credit markets. The typical interpretation of the classical view of a government Lender of Last Resort is based on Bagehot (1873) and maintains that discount-window lending should be done at a penalty rate that is, an interest rate higher than the current interbank market interest rate (although recently challenged by Goodhart, eg. Goodhart (1999)). Dealing with the recent financial crisis, however, the FED has lent funds to institutions in distress under the current market rate. For instance, it has been pointed out that the FED charged the AIG interest rates less than 12%, when similar (near-bankruptcy) quality paper in the market was commanding yields in excess of 17%. Although the difference of illiquidity from insolvency is subtle, at a fundamental level liquidity is related to the "speed" at which assets can be brought into the market while maintaining a substantial part of their value (Keynes (1930): "Bills and call loans are more liquid than investments, i.e., more certainly realisable at short notice without a loss (...)", Hicks (1962)). It is natural to think of the issue of a government as a Lender of Last Resort in a framework that allows for differential ability to bring assets to the market.

In the present paper I am trying to examine the role of the government as a Lender of Last Resort to the financial system in a general equilibrium frame-

work and to find the effects of the provision of the service on the aggregate activity. Traditionally the study of the Lender of Last Resort has been made in a partial equilibrium framework with the understanding that providing the service to individual institutions may have important implications for the function of the aggregate financial system through networks effects among individual financial institutions. In this approach, a recognition of the macroeconomic consequences of the Lender of Last Resort service has been often expressed as the "Too Big To Fail" argument that is, failing of large financial institutions through interconnections of individual banks may cause a precipitate a financial crisis on the aggregate level. In this paper I am putting the question of aggregate consequences of the presence of a government lender at the center of attention considering the case of a representative bank. The objective is to study the mechanism through which the Lender of Last Resort works and identify variables that condition its effectiveness at the aggregate level.

Gertler and Kiyotaki, (2010) present an macroeconomic model of the government as a credit supplier to the financial system, in which credit rationing of financial institutions, due to the possibility of default and inalienability of the human capital of the banker, may create a role for government credit for stabilizing the economy. As such the nature of the credit rationing in this model is not particular to the financial intermediation system. The effectiveness of the government credit-supply mechanism in the model of Kiyotaki and Gertler is based on the superiority of the government technology in monitoring loanable funds offered to banks compared to that of the interbank market. The nature of credit rationing in this paper captures a particular feature of the financial system, the lack of trust, which exacerbates the effects of adverse shocks in the

net worth of the borrower in his ability to obtain funds, however, the friction is not peculiar to the financial intermediation system. As such, the approach does not take into account the particular features of banking and it does not provide a general framework for understanding the role of a Lender of Last Resort.

In the present paper, I investigate the importance of an alternative channel, which puts higher emphasis on a particular feature of the financial intermediation system: the transformation of maturities of assets made by banks, in their role of providing insurance to households against liquidity shocks and consider the government as a credit provider in this respect. The government supply of credit facilitates the insurance provision role of the banking system and it changes the allocation of assets in the banks' portfolios.

I introduce a government sector (Central Bank) in the model of Bernanke and Gertler (1987) to examine the liquidity-provision role of the government. In an economy in which financial intermediaries transform liquid assets (demand deposits) to illiquid assets (long-term loans), the mismatch between maturities of intermediaries' assets and liabilities gives rise to an endogenous liquidity constraint. In the constrained equilibrium of the economy, I consider the effect upon the size of long-term illiquid investment of the government dispatching liquidity at the discount window and the question of the self-sustainability of the government policy.

The contribution of the paper has a two-fold nature: (a) a methodological nature in that it proposes a general equilibrium framework of the government as a Lender of Last Resort, being explicit about the role that financial intermediaries play in the economic environment, so identifying a different mechanism

through which the Lender-of-Last-Resort function matters for aggregate allocations; in particular, the mechanism does not rely on the superior monitoring technology of government in comparison to the one of the financial intermediation system (interbank market for funds). (b) a practical nature, in that it makes an attempt at quantifying the effect of the presence of a Lender of Last Resort on the allocation of investment in the short- and the long-run equilibrium. It also revisits the question of the size of the interest rate at which the government lends to bank at the discount window (the possibility of a "penalty" rate) and of the self-sustainability of the government credit policy.

2 The Model

2.1 The Physical Setup

There are three types of agents in the economy: households, bankers and a government. There is a physical good which takes the form of an endowment and a consumption good. The consumption good is the output of production, it is perishable (it must be immediately consumed after it is produced) and its the only input to production is the endowment good. The endowment good may be stored for one period and then consumed, or invested in a project which yields a random output two periods later. The former technology is referred to as the liquid technology and latter as the illiquid technology. Without screening and monitoring the quality of risky investment projects cannot be evaluated ex ante and their return cannot be observed ex post. Hence there is a role for financial intermediaries in the economy.

2.1.1 Households

There are overlapping generations of consumers of a constant measure 1. Each consumer lives for three periods. A consumer born in period t receives endowment W_t units of goods at date t and consumes either in period $t+1$ (a type- I consumer), or in period $t+2$ (a type- II consumer). A consumer has an exogenous probability of being a type- I consumer equal to α . Consumers allocate their endowments to storage, S_t^h , and deposits at banks, D_t , before they learn their consumption profile type. Let the (ex post) date- $t+1$ and date- $t+2$ utilities of the consumer be: $u(c_{t+1}^I) = \log c_{t+1}^I$ and $u(c_{t+2}^{II}) = \log c_{t+2}^{II}$. The ex post budget constraint of a type- I household born in period t is given by:

$$c_{t+1}^I = R_t^1 D_t + R_t^s S_t^h \quad (1)$$

where R_t^1 is the gross rate of return on demand deposit for a consumer born in period t and withdrawing deposits in period $t+1$ (early-type of consumer) and R_t^s is the gross rate of return on storage.

The ex post constraint a type- II household born in period t is given by:

$$c_{t+2}^{II} = R_t^2 D_t + (R_t^s)^2 S_t^h \quad (2)$$

where R_t^2 is the gross rate of return on consumer deposits for a household born in period t and withdrawing deposits in period $t+2$ (late-type of consumer).

Households' endowments are subject to lump-sum government taxation, X_t , so that the asset allocation of consumers' wealth satisfies:

$$D_t + S_t^h = W_t - X_t \quad (3)$$

2.1.2 Banks

Bankers are infinitely-lived, risk-neutral, and have a constant measure of 1. A banker has an endowment of $W_{b,t}$ units of the endowment good in period t . Bankers sell liabilities (deposits) against endowments in a retail deposit market to consumers and in a wholesale deposit market to other banks. Bankers decide on how to allocate the amount of endowments in their possession between long-term production and storage. Through the long-term, risky technology, investment of I_t units of endowments in period t yields the random gross return of $\tilde{R}_{t+2}I_t$ units of consumption in period $t + 2$. Through the storage technology, investment of S_t^b units of endowment in period t , yields the certain gross return of $R^s S_t^b$ units of consumption in period $t + 1$. In the wholesale deposit market investment of $T_{b,t}$ units of endowment in period t yields the certain gross nominal return of $R_t T_{b,t}$ units of consumption in period $t + 2$.

Investing in the risky technology entails a fixed cost for discerning viable projects and for observing the realization of the random return. Bankers own the monitoring and auditing technology. All viable projects have the same ex-ante distribution of returns. Payment of the cost gives a bank access to a limited pool of projects; there is a restriction on the number of risky projects a banker can undertake. The realizations of returns to the total holdings of risky assets among different bankers are identical; so one can speak of a representative bank.

If the banker undertakes I risky projects the monitoring and auditing cost is δI units of endowments. Let L_t be the endowments a banker borrows at period t , S_t^b the endowments the bank invests in the storage technology, and $h(I_t)$ the total cost of investing in I_t risky projects. Let B_t ($B_t > 0$) be the

amount of credit supplied by the government to banks in period t redeemed in period $t + 1$ for an amount $R_t^w B_t$. The banker's resource constraint in period t is:

$$L_t + W_{b,t} = h(I_t) + S_t^b \quad (4)$$

$$h(I_t) = I_t + \delta I_t \quad (5)$$

Let D_t be the banker's new flow of consumer deposits issued at date t ; they can be redeemed on demand at date $t + 1$, or $t + 2$. Let T_t be the liabilities of a banks issued at date t in the wholesale deposit market. Let $T_{b,t}$ be a bank's holdings of deposits in other banks made in period t . The flow of the banker's new deposits is:

$$L_t = D_t + (T_t - T_{b,t}) \quad (6)$$

Let \tilde{R}_t be the gross date- t return on a banker's risky investments. Assume: $\tilde{R}_t \in [R^l, R^h]$. Let \bar{R} be the mean of \tilde{R} and β the bankers' discount factor. Assume:

$$R^l < R_t^s$$

$$\beta \bar{R} / (1 + \delta) > R_t^s$$

Let R_t^1 and R_t^2 be the gross returns on demand deposits issued at date t and redeemed at dates $t + 1$ and $t + 2$. Let $\tilde{\pi}_t$ be the banker's profit in period t . It is:

$$\begin{aligned} \tilde{\pi}_t = & \tilde{R}_t I_{t-2} + R_{t-1}^s S_{t-1}^b - \alpha R_{t-1}^1 D_{t-1} - (1 - \alpha) R_{t-2}^2 D_{t-2} \\ & - R_{t-2} (T_{t-2} - T_{b,t-2}) - R_{t-1}^w B_{t-1} \end{aligned} \quad (7)$$

The profit in period t is comprised by the returns to assets that is, the return to the illiquid investment, $\tilde{R}_t I_{t-2}$, (made in period $t - 2$) plus the return

to the storage, $R_{t-1}^s S_{t-1}^b$, (made in period $t - 1$), net of the returns to the liabilities that is, the return on the fraction of deposits made in period $t - 1$ and withdrawn in period t , $\alpha R_{t-1}^1 D_{t-1}$, the fraction of deposits made in period $t - 2$ and withdrawn in period t , $(1 - \alpha) R_{t-2}^2 D_{t-2}$, the net return on negative positions in the time-deposit market (taken in period $t - 2$), $R_{t-2}(T_{t-2} - T_{b,t-2})$, and the return on the discount-window loans made in period $t - 1$, $R_{t-1}^w B_{t-1}$.

The financial arrangement based on the demand-deposit contract offered by the bank may be susceptible to bank runs. A necessary condition for a bank run not to occur is that the bank at some date does not have at its disposal a sufficient amount of the consumption good to cover total liabilities. Hence, a non-run equilibrium (non-autarky equilibrium), must necessarily satisfy a liquidity requirement: the banker chooses asset holdings and accepts liabilities in a way that meet obligations at every date and for any realization of returns to the risky investment. That is, the following constraint, pertaining in period $t + 1$, must hold for all $t > 0$:

$$R^l I_{t-1} + R_t^s S_t^b + B_{t+1} \geq \alpha R_t^1 D_t + (1 - \alpha) R_{t-1}^2 D_{t-1} + R_{t-1}(T_{t-1} - T_{b,t-1}) \quad (8)$$

At date $t + 1$, the sum of the minimal return on the illiquid investment (made in period $t - 1$), $R^l I_{t-1}$, the return to storage, and loans at the discount window, B_{t+1} , must be greater or equal to the sum of returns on the fractions of deposits made in period t and in period $t - 1$ and withdrawn in period $t + 1$ plus the return on the net negative position in the time-deposits market taken in period $t - 1$.

Borrowing funds from the government at the discount window has the following features: a) discount-window loans to banks made in period $t + 1$ help relax

the liquidity constraint of the same period that is, they constitute a form of emergency loans to the bank. so that the bank meets its maturing obligations.

b) The discount-window loans taken in period $t + 1$ are an obligation for the bank that must be repaid in period $t + 2$, however we they do not enter the liquidity constraint of period $t + 2$ as a liability item that has to be covered by assets; this implies the bank could possibly roll over the loan that has taken at the discount window.

c) The discount-window loans are not used to finance acquisition of assets, i.e. illiquid investment or storage. In this respect, the present model is a model of government as supplier of the lender-of-last-resort function, rather than a a model of government credit policy.

2.1.3 The Government Behavior

The government may impose taxes and transfers on newly-born households and provide a lender-of-last-resort service to banks. Taxation is lump-sum and the government budget is balanced. At date t the government imposes net taxes X_t from the newly-born agents to finance the net loan-supply operation of the same period. The government's period- t expenses consist of the exogenous government stream \bar{g}_t and the loans made to the bank at the discount window B_t . The period- t revenues for the government come from taxation and the repayment on discount-window loans made in period $t - 1$, $R_{t-1}^w B_{t-1}$, where R_t^w is the gross rate of return for loans made at the discount window. If the revenues from loan repayments exceed the period's expenses, the difference is given to newly-born households as positive transfers. The period- t government's budget constraint for period t are:

$$R_{t-1}^w B_{t-1} + X_t = \bar{g}_t + B_t, \text{ all } t. \quad (9)$$

2.2 The Equilibrium

I consider an equilibrium in which all investment in the storage technology is made by the banks that is, $S_t^h = 0$ at all t .

Definition 1 *An equilibrium with zero consumer storage is a set of sequences of allocations variables $\{c_{t+1}^I, c_{t+2}^{II}, I_t, D_t, S_t^h, S_t^b, T_t, T_{b,t}, B_t, X_t, W_t, W_{b,t}\}_{t=1}^{\infty}$, prices $\{R_t, R_t^1, R_{t+2}^2, R_t^s, R_t^w\}_{t=1}^{\infty}$, and Lagrange multipliers $\{\lambda_t\}_{t=1}^{\infty}$, such that they solve the households' optimization problem, the bank's optimization problem and satisfy the government's budget constraint (9) and the market clearing conditions for the household deposits: $D_t = W_t - X_t$ (10), interbank deposits: $T_t - T_{b,t} = 0$ (11), investment in storage: $S_t = S_t^b$ (12), and the consumption good: $\alpha c_t^I + (1 - \alpha)c_t^{II} = \tilde{R}_t I_{t-2} + S_{t-1}$ (13).*

2.2.1 Characterization of the Equilibrium

2.2.1.1 The Equilibrium of the Bank The banker chooses sequences $\{I_t, D_t, S_t^b, T_t, T_{b,t}, B_t\}_{t=1}^{\infty}$ to maximize expected discounted profits:

$$E_0\{\sum_{t=1}^{\infty} \beta^t \tilde{\pi}_t\}$$

subject to the definition of profits, (7), the balance sheet and flow-of-funds constraints (4), (5), (6) the liquidity constraint (8), the initial values for illiquid investment, storage, consumer and interbank deposits and discount-window lending, $I_{-1}, I_0, S_0^b, D_{-1}, D_0, T_{-1}, T_0, T_{b,-1}, T_{b,0}, B_0$ and the terminal value of the Lagrange multiplier on the liquidity constraint. The problem of the bank is presented in the Appendix.

Let $\beta^{t+1}\lambda_t$ be the present (period- t) shadow value of the liquidity constraint of period $t + 1$. The following conditions characterize an interior solution to the bank's optimization problem with respect to the choices of I_t, S_t^b, T_t (and $T_{b,t}$), B_t , (with D_t substituted in terms of variables using the balance-sheet relations).

$$\frac{\beta}{1+\delta}(\bar{R} + \lambda_{t+1}R^l) = (1 + \lambda_t)\alpha R_t^1 + \beta(1 + \lambda_{t+1})(1 - \alpha)R_t^2 \quad (14)$$

$$(1 + \lambda_t)R_t^s = (1 + \lambda_t)\alpha R_t^1 + \beta(1 + \lambda_{t+1})(1 - \alpha)R_t^2 \quad (15)$$

$$\beta(1 + \lambda_{t+1})R_t = \alpha(1 + \lambda_t)R_t^1 + \beta(1 - \alpha)(1 + \lambda_{t+1})R_t^2 \quad (16)$$

$$\beta R_t^w = \lambda_{t-1} \quad (17)$$

The optimality condition with respect to the investment in the illiquid project (eq. (14)) states that the total expected marginal benefit of investing in the illiquid project (left side) equals the total expected marginal benefit of the investment, in terms of period $t + 1$ consumption (right side). The total expected marginal benefit is comprised by two components: the direct expected return on the projects, \bar{R} , and the indirect expected benefit of having more liquidity two period ahead, $\lambda_{t+1}R^l$, the latter component being the minimal return a liquidity-constrained bank can guarantee its creditors in period $t + 2$. The expected marginal benefit is deflated by the marginal cost of investment in terms of the consumption good, $(1 + \delta)$ and the discount factor β to period- $(t + 1)$ values. The total expected marginal cost of investing in the illiquid project equals the total expected cost of raising one additional unit of deposits for financing investment. The total cost is comprised by the direct cost of raising one unit of deposits, $\alpha R_t^1 + \beta(1 - \alpha)R_t^2$, and the indirect cost, $\lambda_t\alpha R_t^1 + \beta\lambda_{t+1}(1 - \alpha)R_t^2$. The direct cost equals the expected return on one unit of demand deposits raised in period t and liquidated in period $t + 1$, αR_t^1 ,

plus the expected return on one unit of demand deposits raised in period t and liquidated in period $t + 2$, discounted to period $t + 1$, $\beta(1 - \alpha)R_t^2$. The indirect cost equals the expected return on one unit of demand deposits raised in period t and liquidated in period $t + 1$, $\lambda_t\alpha R_t^1$, for a bank that is liquidity-constrained in period $t + 1$, plus the expected return on one unit of demand deposits raised in period t and liquidated in period $t + 2$, discounted to period $t + 1$, $\beta\lambda_{t+1}(1 - \alpha)R_t^2$, for a bank that is liquidity constrained in period $t + 2$.

The optimality condition with respect to storage (eq. (15)) states that the total expected benefit of investing one unit of endowments in storage, $(1 + \lambda_t)R_t^s$, equals the total cost (right side) of financing it through consumer deposits. The total expected marginal benefit is comprised by the direct benefit, the rate of return to storage, 1, plus the indirect benefit of having one additional unit of liquidity in period $t + 1$, for a liquidity- constrained bank, λ_{t+1} .

The optimality condition with respect to time deposits (eq. (16)) states the total expected marginal benefit of making (receiving) one unit of time deposits (left side) equals the total expected marginal cost of raising financing it through consumer deposits (right side). The total expected marginal benefit equals the direct benefit of the rate return on time deposits, R_t , plus the indirect benefit of having one more unit of liquidity in period $t + 2$, for a liquidity-constrained bank, $\lambda_{t+1}R_t$. This is the bank's non-arbitrage condition, saying that the bank does not have a motive for taking an infinitely long, or infinitely short, position in any of the two markets, time and consumer deposits.

The optimality condition with respect to loans at the discount window (eq. (17)) says that the marginal cost of accepting a unit of loans at the discount

window paid off in period $t + 1$ (made in period t) discounted to date t , βR_t^w , must equal the Lagrange multiplier of the date- t constraint that is, the value of liquidity at date t , λ_{t-1} .

2.2.1.2 The Equilibrium of the Household The bank will be indifferent between facing consumers withdrawing early and late, (R_t^1, R_t^2) , if the non-arbitrage condition between returns to demand deposits and R_t is satisfied. As the bank provides liquidity insurance to households, the equilibrium menu of returns on consumer deposits, (R_t^1, R_t^2) implements the constrained-optimal allocation of consumption. That is, the optimal menu of returns maximizes the representative household's expected utility subject to the household's ex post budget constraints (eq. (1) and (2)), the no-consumer-storage condition (eq. (12)) and the bank's non-arbitrage condition (eq. (16)). The consumer's expected utility maximization problem at date t is given by:

$$\max E\{U(c_{t+1}^I, c_{t+2}^{II})\} = \alpha \log c_{t+1}^I + \rho(1 - \alpha) \log c_{t+2}^{II}$$

$$\text{s.t. } c_{t+1}^I = R_t^1(W_t - X_t)$$

$$c_{t+2}^{II} = R_t^2(W_t - X_t)$$

$$\beta(1 + \lambda_{t+1})R_t = \alpha(1 + \lambda_t)R_t^1 + \beta(1 - \alpha)(1 + \lambda_{t+1})R_t^2$$

The following lemmas give the equilibrium returns at a constrained equilibrium.

Lemma 2 *The equilibrium returns on deposits, (R_t^1, R_t^2, R_t) are given by:*

$$R_t^1 = \frac{1}{\alpha + \rho(1 - \alpha)} R_t^s \quad (18)$$

$$R_t^2 = \frac{\rho}{\beta} \left[\frac{1}{\alpha + \rho(1 - \alpha)} \right] \left(\frac{1 + \lambda_t}{1 + \lambda_{t+1}} \right) R_t^s \quad (19)$$

$$R_t = \left(\frac{1}{\beta}\right)\left(\frac{1 + \lambda_t}{1 + \lambda_{t+1}}\right)R_t^s \quad (20)$$

PROOF. In the Appendix.

If the economy is in a Laissez-Faire equilibrium in period t , government taxation and transfers to households and discount-window loans to banks are zero that is, $X_t = 0$ and $B_t = B_{t-1} = 0$. In a Laissez-Faire constrained economy the law of motion of the investment is determined by the dynamics of the wealth of households and banks under the binding liquidity constraint.

From the bank's balance sheet and the flow-of-funds constraint, (eq. (4), (5) and (6)), the bank's investment in storage is given by:

$$S_t^b = W_{b,t} + D_t - (1 + \delta)I_t \quad (21)$$

Substituting for S_t^b from eq.(21), the deposit-market clearing condition, $D_t = W_t$ (eq. (10)), the expressions for R_t^1 and R_t^2 (eq (18) and (19)) into the liquidity constraint, and using the expression for R_t (eq. (20)), we have that at a Laissez-Faire constrained equilibrium, the level of investment, I_t^{LF} , is given by:

$$I_t^{LF} = \frac{1}{(1 + \delta)R_t^s} \left\{ R^l I_{t-1} + R_t^s W_{b,t} + \frac{\rho(1 - \alpha)}{\alpha + \rho(1 - \alpha)} R_t^s W_t - \frac{\rho(1 - \alpha)}{\alpha + \rho(1 - \alpha)} R_{t-1} W_{t-1} \right\} \quad (22a)$$

The dynamics of investment depend on the bank's endowments, consumers' endowments and the interbank rate of interest.

Repeating as in the derivation of eq. (22a), substituting for the deposit-market clearing condition, $D_t = W_t - X_t$ (eq. (3) with $S_t^h = 0$), and setting $B_t =$

$X_t, B_{t-1} = X_{t-1}$, the level of investment at a constrained equilibrium with government intervention, I_t^{GI} , is given by:

$$\begin{aligned}
I_t^{GI} = I_t^{LF} + & \left[\frac{1}{(1 + \delta)R_t^s} \right] \cdot \\
& \left\{ B_{t+1} - \frac{\rho(1 - \alpha)}{\alpha + \rho(1 - \alpha)} R_t^s B_t \right. \\
& \left. - \frac{\rho(1 - \alpha)}{\beta[\alpha + \rho(1 - \alpha)]} \left(\frac{1 + \lambda_t}{1 + \lambda_{t+1}} \right) R_{t-1}^s B_{t-1} \right\}
\end{aligned} \tag{22b}$$

From the bank's non-arbitrage conditions with respect to interbank deposits (eq. 16) and the illiquid asset (eq. 14), it is:

$$R_t(1 + \lambda_{t+1}) = \left(\frac{1}{1 + \delta} \right) (\bar{R} + \lambda_{t+1} R^l)$$

from which the interbank interest rate R_t can be written as:

$$R_t = \frac{\bar{R} - R^l}{(1 + \delta)(1 + \lambda_{t+1})} + \frac{R^l}{(1 + \delta)} \tag{23}$$

Substituting R_{t-1} from (eq. 23) into the expression for the investment dynamics (eq. 22), an increase in the value of liquidity in period $t + 1$, λ_t , decreases the interbank interest rate in period $t - 1$, R_{t-1} , hence the demand deposit rate offered in period $t - 1$ to late withdrawers, R_{t-1}^2 , withdrawing in period $t + 1$. With lower needs for consumption good in period $t + 1$, the bank invests more in the illiquid asset in period t . The opposite happens with an increase in the value of liquidity in period t , λ_{t-1} , which increases R_{t-1} and creates a higher need for consumption goods in period $t + 1$, hence decreases the investment in the illiquid asset, I_t .

From the non-arbitrage conditions for illiquid assets (eq. 14) and storage (eq.

15), it holds:

$$\left(\frac{\beta}{1+\delta}\right)(\bar{R} + \lambda_{t+1}R^l) = R_t^s(1 + \lambda_t)$$

from which we obtain:

$$\lambda_{t+1} = \frac{(1+\delta)R_t^s}{\beta R^l} \lambda_t + \frac{R_t^s(1+\delta) - \beta \bar{R}}{\beta R^l} \quad (24)$$

From the assumptions about the asset returns, we have that the following relations hold: $[(1+\delta)R_t^s]/(\beta R^l) > 1$ and $[R_t^s(1+\delta) - \beta \bar{R}] < 0$.

The dynamics of the system are given by the system of equations (22) and (24) with the initial conditions for investment, I_0 given, and the terminal condition for the Lagrange multiplier, λ_∞ given.

2.2.2 The Steady-State Equilibrium

Definition 3 *A steady-state equilibrium with zero consumer storage is an equilibrium with zero consumer storage with constant prices: $R_t = R$, $R_t^1 = R^1$, $R_t^2 = R^2$, $R_t^s = R^s$, $R_t^w = R^w$, endowments: $W_t = W$, $W_{b,t} = W_b$, investment in the illiquid and liquid assets: $I_t = I$, $S_t^b = S^b$, government loans and taxes: $B_t = B$, $X_t = X$, and Lagrange multipliers: $\lambda_t = \lambda$, for all t .*

There may be several steady-state equilibria, as shown in Bernanke and Gertler (1987). Here, I focus attention on the case of a steady-state equilibrium with a binding liquidity constraint and positive investment in both the illiquid asset and storage.

A steady-state equilibrium can be constructed as follows: from eq. (24), the steady-state Lagrange multiplier satisfies:

$$\lambda = \frac{R^s(1+\delta) - \beta \bar{R}}{\beta R^l - R^s(1+\delta)} > 0$$

From eqs. (18), (19), and (20), we have:

$$R = \left(\frac{1}{\beta}\right)R^s$$

$$R^l = \frac{1}{\alpha + \rho(1 - \alpha)}R^s$$

$$R^2 = \frac{\rho}{\beta} \left[\frac{1}{\alpha + \rho(1 - \alpha)} \right] R^s$$

The steady-state equilibrium storage is found from eq. (21) as:

$$S^b = W + W_b - (1 + \delta)I$$

From the binding liquidity constraint (eq. (22)) and using eq. (20) to substitute for R , the steady-state value of the illiquid investment, I , satisfies:

$$\begin{aligned} [(1 + \delta)R^s - R^l]I = R^s \{ W_b - \left[\frac{\rho(1 - \alpha)}{\alpha + \rho(1 - \alpha)} \right] \left(\frac{1 - \beta}{\beta} \right) W \} \\ + \left\{ 1 + R^s \left(\frac{1 - \beta}{\beta} \right) \left[\frac{\rho(1 - \alpha)}{\alpha + \rho(1 - \alpha)} \right] \right\} B \end{aligned}$$

Given that $(1 + \delta)R^s - R^l > 0$, the following condition is necessary and sufficient for investment in the illiquid asset to be positive:

$$\frac{W_b}{W} > \left(\frac{1 - \beta}{\beta} \right) \left[\frac{\rho(1 - \alpha)}{\alpha + \rho(1 - \alpha)} \right] \left(1 - \frac{B}{W} \right) - \left(\frac{1}{R^s} \right) \frac{B}{W}$$

The expression on the right side of the equation is the minimal ratio of bank capital to consumer endowments. The presence of government lending to banks reduces the minimal size of bank capital that is consistent with an equilibrium with positive investment, for a given size of consumer endowments.

2.2.3 *The Dynamic Behavior of the Equilibrium*

The dynamic behavior of equilibrium is given by the dynamics of the Lagrange multiplier (eq. (24)) and the dynamics of the illiquid investment (eq. (22b), or eq. (22a) and eq. (23)) with the initial condition for investment and terminal condition for the Lagrange multiplier. As investment is a backward-looking variable and the Lagrange multiplier forward-looking, the system has global convergence to the steady-state equilibrium. Moreover, explosive paths for the Lagrange multiplier violate a non-Ponzy-scheme condition (as shown by Bernanke and Gertler). Along such a path, we have: $\lambda_{t+1} > \lambda_t$, so that eq. (20) implies: $\beta R_t = \left(\frac{1+\lambda_t}{1+\lambda_{t+1}}\right)R_t^s < R_t^s$. In such a case a storage intermediary (with zero investment in the illiquid technology) might play a Ponzi scheme, borrowing at date t in the interbank market at the interest rate R_t and investing in storage for return R_t^s . The date- $t + 1$ net value of this scheme is: $R_t^s - \beta R_t > 0$, which would imply infinite profits from following this practise at infinite amounts.

2.3 *Credit Policy*

2.3.1 *The Effectiveness of Credit Policy*

Government in the liquidity-constrained economy may have a stabilizing role in the economy providing liquidity to banks facing a binding liquidity constraint. Although not modeled explicitly in this paper, there might several sources of shocks in the system: shocks in the distribution of returns to the illiquid asset or storage, in the endowments of consumers and banks, and in the probabilities of early-versus-late consumption.

There are two concepts of government stabilization after the incidence of a shock. In the case of a temporary shock the economy follows a path returning the steady-state equilibrium. Temporary government intervention might have the effect of smoothing the cycle that is, making the economy return to the steady state faster. Moreover, as in the case of a large adverse shock in the endowments of banks, the Laissez-Faire financial system may close down. The ratio of bank endowments to consumer endowments may fall below the minimal necessary for positive illiquid investment. In such case, government lending may prevent the economy from temporarily falling into autarky. A similar effect of the government credit policy holds for the long run too. Associated with a certain amount of steady-state government loans given bank endowments there is a certain size of illiquid investment and a certain minimal amount of bank-to-households endowments consistent with positive long-term investment.

In this paper, I consider a non-active government of supplying emergency loans to liquidity-constrained banks at the discount window at an amount decided by the demanding bank, given the shadow value of liquidity, which determines the interest rate at the discount window. I consider the effects of an increase in the loans at the discount window with the shadow value of liquidity remaining constant.

As the amount of loans is decided by the bank, rather than the government, a word of explanation may be due as to the concept of policy experiment. The effect of a marginal change in the amount of loans at the discount window measures the readjustment of the bank's portfolios between illiquid investment and storage, under a fixed discount-window rate, for a liquidity-constrained bank. A bank raising an additional unit of loans at the discount window at the be-

ginning of the following period is able to invest less in the storage technology today and increase investment in the illiquid asset. The effect of such reallocation between investing in storage and borrowing at the discount window measures the effectiveness of the Lender-of-Last-Resort policy. This may be interpreted as the effect on investment of the willingness of the government to provide an additional unit of credit.

We can distinguish between a one-time and a permanent increase in the supply of loans to banks. From the equation of investment dynamics for the constrained economy (eq. (22b)), the effect of a one-time government loan on the size of illiquid investment is given by:

$$\frac{\partial I_t}{\partial B_{t+1}} = \frac{1}{(1 + \delta)R_t^s}$$

For $\delta \approx 0$ and $R_t^s \approx 1$, investment in the illiquid assets in period t raises almost one-to-one with an increase in the supply of government loans.

In the steady-state equilibrium, the effect of a permanent increase in the discount-window loans on investment is given by:

$$\frac{\partial I}{\partial B} = \left[\frac{1}{(1 + \delta)R_t^s - R^l} \right] \left\{ 1 + R^s \left(\frac{1 - \beta}{\beta} \right) \left[\frac{\rho(1 - \alpha)}{\alpha + \rho(1 - \alpha)} \right] \right\}$$

2.3.2 *The Bagehot Rule*

Although there is no concept of incentives and penalties in the economy, it might make sense to consider the difference between the interest rate at the discount window and the interest rate a liquidity-constrained bank could raise through the interbank market, if such a short-term interbank interest rate were available. For simplicity, I will focus on the case of a steady-state equilibrium.

From the non-arbitrage conditions with respect to storage and interbank deposits (eqs. (15) and (16)), it holds:

$$\beta R_t = \left(\frac{1 + \lambda_t}{1 + \lambda_{t+1}} \right) R_t^s$$

In a steady-state equilibrium, the above relation gives:

$$\beta R_t = R_t^s$$

From the non-arbitrage condition with respect to discount-window loans, (eq. (17)) and the steady-state expression for the Lagrange multiplier, the expression for the "penalty" rate is given by:

$$R^w - R^s = \frac{1}{\beta} \lambda - R^s = \frac{(1 + \delta)(1 + \beta R^s)R^s - \beta(\bar{R} + R^s R^l)}{\beta[\beta R^l - R^s(1 + \delta)]}$$

The equilibrium of the economy may exhibit a positive or negative "penalty" rate. As the cost of borrowing at the discount window reflects the shadow value of liquidity for a liquidity-constrained bank, there is a "penalty" rate if the shadow value of the liquidity is sufficiently high. The steady-state value of liquidity depends on the parameters characterizing returns to the illiquid asset. As the denominator of the fraction is negative, a positive "penalty" rate is more probable when \bar{R} takes greater values (eg. for $\beta \approx 1$, $R^s \approx 1$, $\delta \approx 0$, $R^l \approx 0$, it is: $R^w - R^s = \bar{R} - 2$).

2.3.3 *The Self-Sustainability of Credit Policy*

Regarding the issue of the "self-financing" of the government lender-of-last-resort function that is, the possibility that the bank does not need to resort to taxation to finance its lending function, it might be useful to consider the case

that the government borrows to finance the discount window loans. When the government can issue its own liabilities to a subset of the banks to finance the loan operation to other banks, it should offer the interest rate on storage. In this case, the credit policy is "self-sustainable" under a sufficiently high value of liquidity. When the government policy must be financed through public debt sold to households, the government security will be competing with consumer demand deposits. Households born in period t investing in government debt in period t lose the insurance provided by the deposit contract, having a liquidity surplus in period $t + 1$, if their consumption need materializes in period $t + 2$. So, the government debt must compensate households offering a rate of return greater than the interest rate on demand deposits for the early-type of consumer. In this case, a necessary condition for the lending operation to be self-sustainable is: $R_t^w > R_t^l$, that is:

$$\lambda_{t-1} > \frac{1}{\alpha + \rho(1 - \alpha)} R_t^s$$

Financing the lender-of-last-resort function through public debt sold to financial intermediaries would correspond to a liquidity crisis of individual institutions. Financing the lender-of-last-resort service through issuing public debt to households would correspond to an aggregate liquidity crisis. In both cases the supply of the lender-of-last-resort function during a liquidity crisis is a self-sustainable operation, when the value of liquidity in the period of the loan is sufficiently high.

In the steady-state equilibrium of the constrained economy the above condition is written as:

$$\frac{\beta \bar{R} - R^s(1 + \delta)}{R^s(1 + \delta) - \beta R^l} > \frac{R^s}{\alpha + \rho(1 - \alpha)}$$

showing that self-sustainability is a property of a constrained economy with sufficiently high average returns to the illiquid investment.

3 Conclusions

In a general equilibrium model of financial intermediation in which banks provide liquidity insurance to households, matching the financial needs of entrepreneurs with long-term investment projects and households with random liquidity needs, an equilibrium with an active financial system requires the fulfilment of an endogenous liquidity constraint. The liquidity constraint functions as a credit-rationing constraint in that the size of the financial intermediary's net worth constrains the intermediary's leverage and the amount of illiquid investment that can be made. An exogenous fall in the net worth of banks gives rise to a credit crunch, or liquidity crisis, that depresses the issuance of deposits and investment in long-term projects.

In the liquidity-constrained economy there is potentially a role for the government as a Lender of Last Resort. The government's quick dispatching of liquidity at the discount window is effective in increasing investment in the illiquid asset in both short- and long-term equilibrium. The government's Lender-of-Last-Resort function stabilizes the economy against short-term fluctuations in variables such as the endowments of banks and prevents the financial system from closing down in the case of large adverse shocks, reducing the necessary ratio of bank-to-household wealth that is consistent with an equilibrium with positive investment.

Whether government lending at the discount window involves a "penalty"

rate or not depends on the shadow value of liquidity, with the presence of a penalty rate more likely when the value of liquidity is higher. The Lender-of-Last-Resort operation is a self-sustainable operation when the shadow value of liquidity is sufficiently high.

4 Appendix

The Lagrangian Function for the Bank's Problem

The banker chooses sequences $\{I_t, D_t, S_t^b, T_t, T_{b,t}, B_t\}_{t=1}^{\infty}$ to maximize expected discounted profits:

$$E_0\{\sum_{t=1}^{\infty}\beta^t\tilde{\pi}_t\}$$

subject to:

$$L_t + W_{b,t} = h(I_t) + S_t^b$$

$$h(I_t) = I_t + \delta I_t \quad (5)$$

$$L_t = D_t + (T_t - T_{b,t}) \quad (6)$$

$$\begin{aligned} \tilde{\pi}_t = & \tilde{R}_t I_{t-2} + R_{t-1}^s S_{t-1}^b - \alpha R_{t-1}^1 D_{t-1} - (1 - \alpha) R_{t-2}^2 D_{t-2} \\ & - R_{t-2} (T_{t-2} - T_{b,t-2}) - R_{t-1}^w B_{t-1} \end{aligned} \quad (7)$$

$$R^l I_{t-1} + R_t^s S_t^b + B_{t+1} \geq \alpha R_t^1 D_t + (1 - \alpha) R_{t-1}^2 D_{t-1} + R_{t-1} (T_{t-1} - T_{b,t-1}) \quad (8)$$

$$\text{given } I_{-1}, I_0, S_0^b, D_{-1}, D_0, T_{-1}, T_0, T_{b,-1}, T_{b,0}, B_0$$

The Lagrangian function is given by:

$$\begin{aligned} \Lambda_0 = & E_0 \sum_{t=1}^{\infty} \{ \beta^t \tilde{\pi}_t + \\ & \beta^{t+1} \lambda_t [R^l I_{t-1} + R_t^s S_t^b + B_{t+1} - \alpha R_t^1 D_t - (1 - \alpha) R_{t-1}^2 D_{t-1} - R_{t-1} (T_{t-1} - T_{b,t-1})] \} \end{aligned}$$

Using eqs. (4), (5) and (6), we can express D_t as:

$$D_t = (1 + \delta)I_t + S_t^b - (T_t - T_{b,t}) - W_{b,t}$$

Substituting from the above expression for D_t and D_{t-1} in the Lagrangian function, this can be written:

$$\begin{aligned}
\Lambda_0 = & E_0 \sum_{t=1}^{\infty} \beta^t \{ \tilde{R}_t I_{t-2} + R_{t-1}^s S_{t-1}^b \\
& - \alpha R_{t-1}^1 [(1 + \delta) I_{t-1} + S_{t-1}^b - (T_{t-1} - T_{b,t-1}) - W_{b,t-1}] \\
& - (1 - \alpha) R_{t-2}^2 [(1 + \delta) I_{t-2} + S_{t-2}^b - (T_{t-2} - T_{b,t-2}) - W_{b,t-2}] \\
& - R_{t-2} (T_{t-2} - T_{b,t-2}) - R_{t-1}^w B_{t-1} \} \\
& + E_0 \sum_{t=1}^{\infty} \beta^{t+1} \lambda_t \{ R^l I_{t-1} + R_t^s S_t^b + B_{t+1} \\
& - \alpha R_t^1 [(1 + \delta) I_t + S_t^b - (T_t - T_{b,t}) - W_{b,t}] \\
& - (1 - \alpha) R_{t-1}^2 [(1 + \delta) I_{t-1} + S_{t-1}^b - (T_{t-1} - T_{b,t-1}) - W_{b,t-1}] \\
& - R_{t-1} (T_{t-1} - T_{b,t-1}) \}
\end{aligned}$$

The first-order condition of the maximization problems (for an interior solution) are given by:

$$\begin{aligned}
\frac{\partial \Lambda_0}{\partial I_t} = & \beta^{t+2} \bar{R}_{t+2} - \beta^{t+1} \alpha R_t^1 (1 + \delta) - \beta^{t+2} (1 - \alpha) R_t^2 (1 + \delta) \\
& + \beta^{t+2} \lambda_{t+1} R^l \\
& - \beta^{t+1} \alpha R_t^1 (1 + \delta) \lambda_t - \beta^{t+2} (1 - \alpha) R_t^2 (1 + \delta) \lambda_{t+1} = 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Lambda_0}{\partial S_t^b} = & \beta^{t+1} R_t^s - \beta^{t+1} \alpha R_t^1 - \beta^{t+2} (1 - \alpha) R_t^2 \\
& + \beta^{t+1} \lambda_t R_t^s - \beta^{t+1} \alpha R_t^1 \lambda_t - \beta^{t+2} (1 - \alpha) R_t^2 \lambda_{t+1} = 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Lambda_0}{\partial (T_t - T_b)} = & \beta^{t+1} \alpha R_t^1 + \beta^{t+2} (1 - \alpha) R_t^2 - \beta^{t+2} R_t \\
& + \beta^{t+1} \alpha R_t^1 \lambda_t + \beta^{t+2} (1 - \alpha) R_t^2 \lambda_{t+1} - \beta^{t+2} \lambda_{t+1} R_t = 0
\end{aligned}$$

$$\frac{\partial \Lambda_0}{\partial B_{t+1}} = -\beta^{t+2} R_{t+1}^w + \beta^{t+1} \lambda_t = 0$$

Moving terms involving the returns on consumer deposits to the right side (the marginal cost of raising consumer deposits) in the first three of the above conditions and simplifying the powers of β , the system of first-order conditions can be written as follows:

$$\beta (\bar{R}_{t+2} + \lambda_{t+1} R^l) = \alpha (1 + \delta) (1 + \lambda_t) R_t^1 + \beta (1 - \alpha) (1 + \delta) (1 + \lambda_{t+1}) R_t^2$$

$$\begin{aligned}
(1 + \lambda_t)R_t^s &= \alpha(1 + \delta)(1 + \lambda_t)R_t^1 + \beta(1 - \alpha)(1 + \delta)(1 + \lambda_{t+1})R_t^2 \\
\beta(1 + \lambda_{t+1})R_t &= \alpha(1 + \delta)(1 + \lambda_t)R_t^1 + \beta(1 - \alpha)(1 + \delta)(1 + \lambda_{t+1})R_t^2 \\
\lambda_t &= \beta R_{t+1}^w
\end{aligned}$$

Lemma. The equilibrium returns on deposits, (R_t^1, R_t^2, R_t) are given by:

$$\begin{aligned}
R_t^1 &= \frac{1}{\alpha + \rho(1 - \alpha)} R_t^s \\
R_t^2 &= \frac{\rho}{\beta} \left[\frac{1}{\alpha + \rho(1 - \alpha)} \right] \left(\frac{1 + \lambda_t}{1 + \lambda_{t+1}} \right) R_t^s \\
R_t &= \left(\frac{1}{\beta} \right) \left(\frac{1 + \lambda_t}{1 + \lambda_{t+1}} \right) R_t^s
\end{aligned}$$

PROOF. The Lagrangian function related to the household's optimization problem is given by:

$$\max_{R_t^1, R_t^2} \{ \alpha \log(R_t^1 W_t) + \rho(1 - \alpha) \log(R_t^2 W_t) + \mu [\beta R_t(1 + \lambda_{t+1}) - (1 + \lambda_t)\alpha R_t^1 - \beta(1 + \lambda_{t+1})(1 - \alpha)R_t^2] \}$$

The first-order conditions of the problem give the expressions for R_t^1 and R_t :

$$\begin{aligned}
R_t^1 &= \frac{1}{\mu\beta(1 + \lambda_t)} \\
R_t^2 &= \frac{\rho}{\mu\beta(1 + \lambda_{t+1})}
\end{aligned}$$

That is, we have:

$$R_t^1 = \frac{\beta(1 + \lambda_{t+1})}{\rho(1 + \lambda_t)} R_t^2$$

Substituting for R_t^1 into the non-arbitrage condition between time and demand deposits, we have:

$$R_t^2 = \frac{\rho}{\alpha + \rho(1 - \alpha)} R_t$$

Substituting for R_t^2 into the condition for R_t^1 , we get:

$$R_t^1 = \frac{\beta}{\alpha + \rho(1 - \alpha)} \frac{(1 + \lambda_{t+1})}{(1 + \lambda_t)} R_t$$

Substituting for R_t^1 and R_t^2 into the optimality condition with respect to S_t , $R_t^s(1 + \lambda_t) = (1 + \lambda_t)\alpha R_t^1 + \beta(1 + \lambda_{t+1})(1 - \alpha)R_t^2$, we obtain:

$$R_t^s = \beta\left(\frac{1 + \lambda_{t+1}}{1 + \lambda_t}\right)R_t$$

Solving for R_t and substituting back into the expressions for R_t^1 and R_t^2 , we obtain the result.

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